Example phrases	**Ontology**		**Sketch**	**Tickmarks**	**Entries and Equations**
when, at	Time	T0. Instance	Stopwatch	Angular orientation of stopwatch hand	Time t
during, from		T1. Episode	Stopwatch	Stopwatch hand's initial and final orientations Arc stopwatch hand's tip traces during episode	Duration Δt
at, point, above, under, below, near	Territory	L0. Point	Labeled dot Landmark(s)	Single tickmark for each coordinate axis	Ordered pair
along, path, road, highway, journey, traveled		L1. Path	Labeled lane markers or footprints Landmark(s)	Arrowhead(s) for direction (if available) Representative tickmarks delineating unit segments Tripometer hand's initial & final angular orientations Arc tripometer hand's tip traces while traversing path	Traveled distance d or L Equation of graph of path
surface, area, blanket, face, panel, quilt, floor, wall		L2. Surface	Labeled borders (segment if seen "edge on") Landmark(s)	Representative unit squares (possibly deformed)	Area A Equation of surface Equations of boundaries
solid, sphere, cube, cylinder, prism, box, block, ball		L3. Solid form (with important structure)	Labeled silhouette Belt Landmark(s)	Representative unit cubes	Volume V
box, block, person, car, cart, ball, table, home, building, cliff		L4. Solid form (without important structure)	Labeled stick figure	Unnecessary	Unnecessary
angle, degrees, radians, arc, sweep		L5. Angle	Initial side Final side Nearby landmark(s)	Arc	Angle measure θ
mass, charge, money, dollars, balance, energy, intensity, measured, recorded, registered, reads	Quantitative property	Q1. Unsigned or signed mass, charge, energy, intensity account balance	Container (stick figure of vehicle, person, box, or bag)	Bar chart Graph/plot	Mass m Account or instrument value V etc.
change, increase, decrease		Q2. Changed mass, charge, energy, intensity, account balance	Snapshots of container (stick figure of vehicle, person, box, or bag) changing	On bar chart and/or graph/plot, draw arrow for each change in each asset involved in the transfer (use dashed guidelines if related initial and final bars are not next to each other)	
exchange, transfer, delivered, performed on, given to, taken from		Q3. Transferred mass, charge, energy, account balance	Parcel in midst of transfer, with trailing whooshies	Draw a bar-chart bar for each transferred amount	Each asset change that's part of the transfer has its own table entry
directed quantity, magnitude and direction		Q4. Vector	N/A	Arrow Anchor tail to dot for snapshot/object vector belongs to Scale arrow's length to represent vector's magnitude Tilt the arrow to indicate the vector's direction If direction information is available, use L5. Angle	Vector $\vec{v}$ Magnitude v Components v_x, v_y Direction angle θ
speed, velocity, movement, traveling, fast, slow, direction of motion, walking, running, driving, flying, braking	Rate	R1. Spatial movement	Trailing whooshies	Angular orientation of speedometer's hand Triangle illustrating slope of secant line or tangent line for graph of position or distance vs. time If direction is available, refer to Q4. Vector	Velocity components $v_x = \frac{\Delta x}{\Delta t}$, $v_y = \frac{\Delta y}{\Delta t}$ Speed $v = \frac{d}{t}$
rate, per, every		R2. General tandem change	Small burst of successive frames	Triangle illustrating slope of secant line or tangent line for graph of output vs. input	Rate of change $r = \frac{\Delta y}{\Delta x}$

Example phrases	**Ontology**		**Sketch**	**Tickmarks**	**Entries and Equations**
equals, less than, greater than, is the same as, function of, $y =$, versus, corresponds to, for each, represents, translates to, per	Mathematical abstractions	M1. Relationship	N/A	Graph/plot of output vs. input Input axis Output axis Label graph/plot (If visualizing multiple input variables is difficult, draw various "traces", each of which varies only one of the input variables) If a graph illustrates a relationship sufficiently, rows M3 through M7 need not be individually used.	Equation or inequality x-y table (possibly with a separate row for each operation in the formula for y)
percent		M2. Percent	N/A	Double-number line (physical units on one axis, percentages on parallel axis)	$\frac{\text{part}}{\text{reference}} = \frac{\text{percentage}}{100}$
add, sum		M3. Add	N/A	Head-to-tail addition along real number line	$+$
subtract, difference		M4. Subtract	N/A	Head-to-tail addition along real number line with reversed 2nd arrow	$-$
multiply, product		M5. Multiply	N/A	Box model (factors label sides and product labels area) If numerical values are available, you can simply represent the multiplication by highlighting corresponding tickmarks on a coordinate axis.	$\cdot$ or $\times$
divide, quotient, ratio, fraction		M6. Divide	N/A	Box model (dividend labels area, divisor and quotient label sides) If numerical values are available, you can simply represent the multiplication by highlighting corresponding tickmarks on a coordinate axis.	$\div$, $/$, or $-$
equal, same, equivalent		M7. Equal	N/A	Paired depictions of same numbers of unit steps, unit squares, etc.	$=$
constant, for all	Pattern/prevalence establishment	P1. Sustain, make property shared	(Duplicate)	(Duplicate)	(Duplicate)
if, then, if and only if, necessary and sufficient, implies, requires		P2. If-then statement	N/A	Optional (virtually never used): Truth graph	Table with if and then cells Implication arrows $\Rightarrow$ or $\Leftrightarrow$ Optional: Truth table
claim, according to, believes, argues, consistent with, corrects	Perspective taking	A1. Attribution	Speech or thought bubble	Put tickmarks, axes, and their associated graphical representations inside speech/thought bubbles	They say-I say-Agree? chart (cf. Graff *et al.*)
				For college prob/stat classes: Graph of a Bayesian prior	For college prob/stat classes: Bayesian prior function
find, determine, state, what	Command/request	C1. Find	Usually N/A	Question mark near tickmark/axis system associated with request	Question mark near algebraic symbol(s) associated with request